Further Part 1 - January 2012

(1) a) $\alpha + \beta = -7/2 \qquad \alpha\beta = 4$

b) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-7/2)^2 - 2(4)$

$= 49/4 - 8 = 17/4$

c) $\boxed{\text{SUM}}$ $\quad 1/\alpha^2 + 1/\beta^2 = \dfrac{\beta^2}{\alpha^2\beta^2} + \dfrac{\alpha^2}{\alpha^2\beta^2}$

$= \dfrac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \dfrac{17/4}{4^2} = 17/64$

$\boxed{\text{PRODUCT}}$ $\quad 1/\alpha^2 \times 1/\beta^2 = \dfrac{1}{\alpha^2\beta^2} = \dfrac{1}{(\alpha\beta)^2} = \dfrac{1}{4^2} = 1/16$

$x^2 - \boxed{\text{SUM}}\,x + \boxed{\text{PRODUCT}} = 0$

$\rightarrow x^2 - 17/64\, x + 1/16 = 0$

$64x^2 - 17x + 4 = 0$

(2) a) $\displaystyle\int_8^\infty x^{-2/3}\,dx = \int_8^p x^{-2/3} = \left[3x^{1/3}\right]_8^p$

$= 3\sqrt[3]{p} - 3\sqrt[3]{8}$

As $p \to \infty$, $3\sqrt[3]{p}$ does not converge to a limit

$\therefore \int$ has no finite value

b) $\displaystyle\int_8^\infty x^{-4/3}\,dx = \int_8^p x^{-4/3} = \left[-3x^{-1/3}\right]_8^p$

$= \dfrac{-3}{\sqrt[3]{p}} - \dfrac{-3}{\sqrt[3]{8}}$

$= \dfrac{3}{2} - \dfrac{3}{\sqrt[3]{p}}$

As $p \to \infty$, $\dfrac{3}{\sqrt[3]{p}} \to 0$, so $\int \to 3/2$

③ a) i) $x^2 + 9 = 0 \rightarrow x^2 = -9 \rightarrow x = \pm\sqrt{-9} \rightarrow x =$

ii) $(x+2)^2 + 9 = 0 \rightarrow x + 2 = \pm 3i$

$\rightarrow x = -2 \pm 3i$

b) i) $(1+x)^3 = 1 + 3x + 3x^2 + x^3$

ii) $(1+2i)^3 = 1 + 3(2i) + 3(2i)^2 + (2i)^3$

$= 1 + 6i + 12i^2 + 8i^3$

$= 1 + 6i - 12 - 8i = -11 - 2i$

iii) $z^* - z^3 = (1-2i) - (-11-2i)$

$= 1 - 2i + 11 + 2i$

$= 12$

④ a) $\sum r^2(4r-3) = 4\sum r^3 - 3\sum r^2$

$= n^2(n+1)^2 - \frac{1}{2}n(n+1)(2n+1)$

$= \frac{1}{2}n(n+1)\left[2n(n+1) - (2n+1)\right]$

$= \frac{1}{2}n(n+1)\left[2n^2 + 2n - 2n - 1\right]$

$= \frac{1}{2}n(n+1)(2n^2 - 1)$

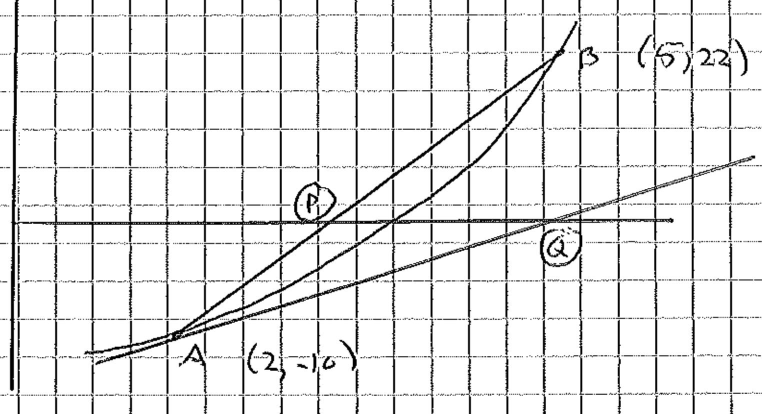
b) $\sum_{20}^{40} = \sum_{1}^{40} - \sum^{19}$

$= \frac{1}{2}(40)(41)(2(40^2)-1) - \frac{1}{2}(19)(20)(2(19^2)-1)$

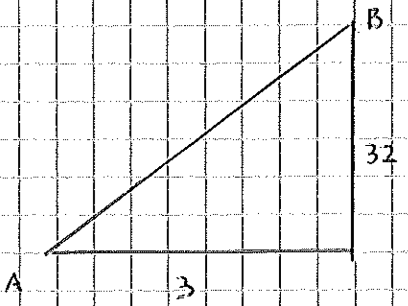
$= 2623180 - 136996$

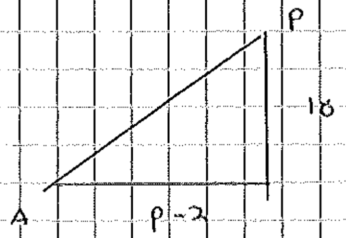
$= 2,486,190$

⑤ a)



ii)





$$\frac{p-2}{10} = \frac{3}{32}$$

$$\Rightarrow p-2 = \frac{30}{32} \quad \Rightarrow p = \frac{30}{32} + 2 = 47/16$$

b) iii) $x_1 = 2$

$y_1 = -10$

$m = 8$

$y - y_1 = m(x - x_1)$

$y + 10 = 8(x - 2)$

$y + 10 = 8x - 16$

At Q, $y = 0 \Rightarrow 10 = 8x - 16$

$8x = 26 \Rightarrow x = 26/8$ or 3.25

(6) a) $\theta = n\pi + a$

Key angle $= \tan^{-1}(1/\sqrt{3}) = \pi/6 = a$

$\Rightarrow x/2 - \pi/4 = n\pi + \pi/6$

$\Rightarrow x/2 = n\pi + 5\pi/12$

$\Rightarrow x = 2n\pi + 5\pi/6$

b) $\tan^2(x/2 - \pi/4) = 1/3$

$\sqrt{}$ $\Rightarrow \tan(x/2 - \pi/4) = \pm 1/\sqrt{3}$

$\therefore \tan(x/2 - \pi/4) = 1/\sqrt{3}$ $\boxed{\text{OR}}$ $\tan(x/2 - \pi/4) = -1/\sqrt{3}$

$\Rightarrow 2n\pi + 5\pi/6$

(from a)

Key angle: $\tan^{-1}(-1/\sqrt{3}) = -\pi/6$

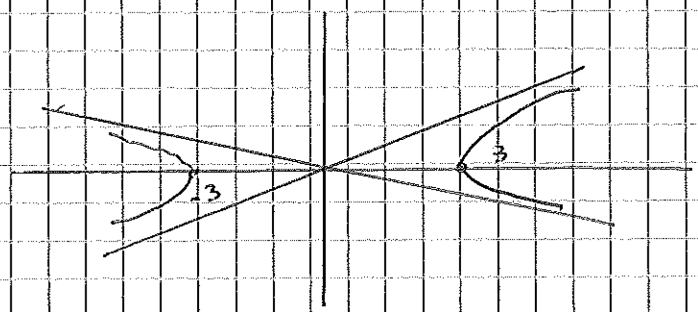
$\Rightarrow x/2 - \pi/4 = n\pi - \pi/6$

$\Rightarrow x/2 = n\pi + \pi/12$

$\Rightarrow x = 2n\pi + \pi/6$

(7) a) $\frac{x}{3} = \pm y$ (from formula book) $\Rightarrow y = \pm \frac{1}{3} x$

b)



Intersection $= (3, 0)$

$(-3, 0)$

c) i) $\frac{(x+3)^2}{9} - y^2 = 1$

ii) $y = x \Rightarrow \frac{(x+3)^2}{9} - x^2 = 1$

$\Rightarrow (x+3)^2 - 9x^2 = 9$

$\Rightarrow x^2 + 6x + 9 - 9x^2 = 9$

$\Rightarrow 8x^2 - 6x = 0$

$\Rightarrow 4x^2 - 3x = 0$

$\Rightarrow x(4x - 3) = 0$

$(0,0)$ $x = 0$ $\quad$ $x = 3/4$ $(3/4, 3/4)$

$y = 0$ $\quad$ $y = 3/4$

d) The curve shifts 3 to the right

$y = x - 3 \Rightarrow y + 3 = x$ $\therefore$ line also shifts 3 to right

$\therefore$ Intersections are 3 to right

$= (3, 0)$ or $(3\frac{3}{4}, \frac{3}{4})$

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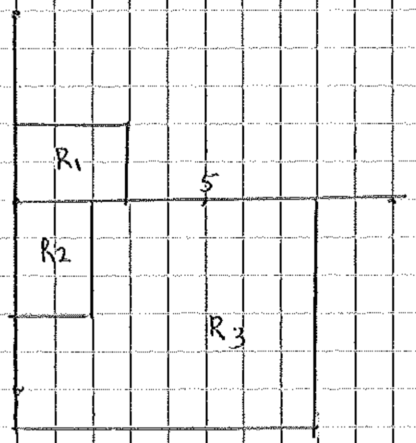


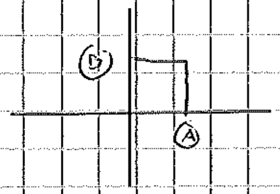
i) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

ii) $\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$

= stretch SF 4 in x

stretch SF 2 in y

b) i) $R_1 \to R_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$



ii) $R_1 \to R_3 = R_3 \times R_2$

$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0 & 4 \\ -2 & 0 \end{pmatrix}$

9) a) Denominator $\to x = 1$

Numerator $\to y = 1/1 \to y = 1$

b) $y = -4x + c \to -4x + c = \frac{x}{(x-1)}$

$\to (-4x+c)(x-1) = x$

$\to -4x^2 + 4x + cx - c = x$

$\to 4x^2 - 3x - cx + c = 0$

$\to 4x^2 - (c+3)x + c = 0$

c) At tangent, $b^2 - 4ac = 0$

$\to (-c-3)^2 - 4(4)(c) = 0$

$\to c^2 + 6c + 9 - 16c = 0$

$\to c^2 - 10c + 9 = 0$

$(c-9)(c-1) = 0 \to c = 9, \; c = 1$

ii) $c = 1$ $\quad 4x^2 + 4x + 1 = 0$

$(2x + 1)(2x + 1) = 0$

$\Rightarrow x = 1/2$

$\Rightarrow y = -4(1/2) + 1 = -1 \quad \Rightarrow (1/2, -1)$

$c = 9$ $\quad 4x^2 - 12x + 9 = 0$

$(2x - 3)(2x - 3) = 0$

$\Rightarrow x = 3/2$

$\Rightarrow y = -4(3/2) + 9 = 3 \quad \Rightarrow (3/2, 3)$